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Holographic renormalization group and conformal anomaly for $\mathbf{AdS_9/CFT_8}$ correspondence

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ABSTRACT

Holographic Renormalization Group (RG) in nine dimensions is considered. The d8 holographic conformal anomaly is found. It should correspond to d8 CFT in $\mathrm{AdS_9/CFT_8}$ correspondence. The comparison of holographic and QFT anomalies in d8 de Sitter space is done. It may give the indication for rigorous $\mathrm{AdS_9/CFT_8}$ correspondence proposal.

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The Hamilton-Jacobi formalism in higher dimensions provides the interesting formulation of holographic RG [1] (for related works, see[2, 3]). This is extremely useful in the study of RG flows[4] (and references therein) from SG side in AdS/CFT correspondence[5]. In particulary, such investigation gives the possibility to use the solutions of multi-dimensional (gauged) supergravity in order to describe RG flows in dual CFT living on the boundary of corresponding AdS spacetime. One of the quantities playing the important role in AdS/CFT correspondence [5] is holographic conformal anomaly [6]. For example, the comparison of holographic and QFT conformal anomalies helps in the explicit identification of boundary CFT[4] with the corresponding SG dual. Moreover, the holographic RG is the only known way to get the non-perturbative conformal anomaly (which is actually impossible to do in frames of usual perturbative QFT). The maximally SUSY Yang-Mills theory is the only exclusion from this rule: here QFT calculation for exact, non-perturbative result is possible.

It is known quite a lot about AdS/CFT correspondence in dimensions below 8, say, about AdS_7/CFT_6, AdS_5/CFT_4 or AdS_3/CFT_2 set-up (see review [8] and refs. therein). The corresponding exact CFTs in dimensions 2,4,6 are explicitly constructed. It would be really unnatural to expect that Nature is not symmetric and that ,say, AdS_9/CFT_8 correspondence and corresponding exact CFT do not occur. Hence, it would be of great interest to extend the corresponding results to higher dimensions: d9, d11 (where M-theory is presumbly residing)[5]. As one step in this direction one can calculate the

[4]The brane quantum gravity presumbly may be included into this identification as next-to-leading effect [7].

[5]Despite some attempts d8 CFT is not constructed yet explicitly. The multiplet of CFT_8 could be one including rank 3 (with 3 vector indeces and maybe anti-symmetric) tensor field. We need to consider what is the low energy effective theory in D7-brane. Let us consider the situation where N branes are piled up. If a (open) string connect two branes, the string corresponds to vector field in the low energy and the field contains N^2 components, which correspond to $\mathcal{N} = 4$ Yang-Mills theory in 4 dimensions. If strings with junction (3 strings with one vertex) connect 3 branes, it corresponds to rank 2 tensor field with N^3 components, which lead to CFT_6. If strings (or maybe strings with two junctions or verteces) connect 4 branes, it would correspond to rank 3 tensor field with N^4 components, which would correspond to CFT_8. Note that the number of the vector indeces corresponds to the directions from one fixed brane to the other branes, then the number of the indeces is the number of the connected branes minus one. Another way of doing things could be to start from d10 supergravity, to compactify it to 8 dimensions and to try to construct something like modified 8d vector multiplet. Clearly, it is not an easy

holographic conformal anomaly in higher dimensions. In the present Letter, starting from the systematic prescription for solving the Hamilton-Jacobi equation (i.e. flow equation) given in [9], we will perform such a calculation in eight dimensions.[6]

First, we briefly review the formulation discussed in [1, 9]. One starts from $d+1$ dimensional AdS-like metric in the following form

$$ds^2 = G_{MN}dX^M dX^N = dr^2 + G_{\mu\nu}(x,r)dx^\mu dx^\nu. \tag{1}$$

where $X^M = (x^\mu, r)$ with $\mu, \nu = 1, 2, \cdots, d$. The action on a $(d+1)$ dimensional manifold M_{d+1} with the boundary $\Sigma_d = \partial M_{d+1}$ is given by

$$\begin{aligned} S_{d+1} &= \int_{M_{d+1}} d^{d+1}x\sqrt{G}(V-R) - 2\int_{\Sigma_d} d^dx\sqrt{G}K \\ &= \int_{\Sigma_d} d^dx \int dr\sqrt{G}\left(V - R + K_{\mu\nu}K^{\mu\nu} - K^2\right) \\ &\equiv \int d^dx dr\sqrt{G}\mathcal{L}_{d+1}. \end{aligned} \tag{2}$$

where R and $K_{\mu\nu}$ are the scalar curvature and the extrinsic curvature on Σ_d respectively. $K_{\mu\nu}$ is given as

$$K_{\mu\nu} = \frac{1}{2}\frac{\partial G_{\mu\nu}}{\partial r}, \quad K = G^{\mu\nu}K_{\mu\nu} \tag{3}$$

In the canonical formalism, $\mathcal{L}_{d+1}$ is rewritten by using the canonical momenta $\Pi_{\mu\nu}$ and Hamiltonian density $\mathcal{H}$ as

$$\mathcal{L}_{d+1} = \Pi^{\mu\nu}\frac{\partial G_{\mu\nu}}{\partial r} + \mathcal{H}\ , \quad \mathcal{H} \equiv \frac{1}{d-1}(\Pi^\mu_\mu)^2 - \Pi^2_{\mu\nu} + V - R\ . \tag{4}$$

The equation of motion for $\Pi^{\mu\nu}$ leads to

$$\Pi^{\mu\nu} = K^{\mu\nu} - G^{\mu\nu}K. \tag{5}$$

task.

[6]The very interesting attempt based on counterterm method to calculate d8 conformal anomaly has been performed in ref.[10]. However, the explicit result was not obtained there. Moreover, it is known [3] that counterterm method in higher dimensions may lead to some ambigious result.

The Hamilton constraint $\mathcal{H}=0$ leads to the Hamilton-Jacobi equation (flow equation)

$$\begin{aligned}
\{S,S\}(x) &= \sqrt{G}\mathcal{L}_d(x) && (6)\\
\{S,S\}(x) &\equiv \frac{1}{\sqrt{G}}\left[-\frac{1}{d-1}\left(G_{\mu\nu}\frac{\delta S}{\delta G_{\mu\nu}}\right)^2+\left(\frac{\delta S}{\delta G_{\mu\nu}}\right)^2\right], && (7)\\
\mathcal{L}_d(x) &\equiv V-R[G]. && (8)
\end{aligned}$$

One can decompose the action S into a local and non-local part discussed in ref.[1] as follows

$$S[G(x)] \;=\; S_{loc}[G(x)]+\Gamma[G(x)], \tag{9}$$

Here $S_{loc}[G(x)]$ is tree level action and Γ contains the higher-derivative and non-local terms. In the following discussion, we take the systematic method of ref.[9], which is weight calculation. The $S_{loc}[G]$ can be expressed as a sum of local terms

$$S_{loc}[G(x)]=\int d^dx\sqrt{G}\mathcal{L}_{loc}(x)=\int d^dx\sqrt{G}\sum_{w=0,2,4,\cdots}[\mathcal{L}_{loc}(x)]_w \tag{10}$$

The weight w is defined by following rules;

$$G_{\mu\nu},\ \Gamma:\text{weight }0\ ,\quad \partial_\mu:\text{weight }1\ ,\quad R,\ R_{\mu\nu}:\text{weight }2\ ,\quad \frac{\delta\Gamma}{\delta G_{\mu\nu}}:\text{weight }d\ .$$

Using these rules and (6), one obtains the equations, which depend on the weight as

$$\begin{aligned}
\sqrt{G}\mathcal{L}_d &= [\{S_{loc},S_{loc}\}]_0+[\{S_{loc},S_{loc}\}]_2 && (11)\\
0 &= [\{S_{loc},S_{loc}\}]_w \quad (w=4,6,\cdots d-2), && (12)\\
0 &= 2\,[\{S_{loc},\Gamma\}]_d+[\{S_{loc},S_{loc}\}]_d && (13)
\end{aligned}$$

The above equations which determine $[\mathcal{L}_{loc}]_w$. $[\mathcal{L}_{loc}]_0$ and $[\mathcal{L}_{loc}]_2$ are parametrized by

$$[\mathcal{L}_{loc}]_0=W\ ,\quad [\mathcal{L}_{loc}]_2=-\Phi R\ . \tag{14}$$

Thus one can solve (11) as

$$V=-\frac{d}{4(d-1)}W^2\ ,\quad -1=\frac{d-2}{2(d-1)}W\Phi\ . \tag{15}$$

Setting $V = 2\Lambda = -d(d-1)/l^2$, where Λ is the bulk cosmological constant and the parameter l is the radius of the asymptotic AdS_{d+1}, we obtain W and Φ as

$$W = -\frac{2(d-1)}{l}, \quad \Phi = \frac{l}{d-2}. \tag{16}$$

To obtain the higher weight ($w \geq 4$) local terms related with conformal anomaly, we introduce a local term $[\mathcal{L}_{loc}]_4$ as follows

$$[\mathcal{L}_{loc}]_4 = XR^2 + YR_{\mu\nu}R^{\mu\nu} + ZR_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}. \tag{17}$$

Here X, Y and Z are some constants determined by (12). The calculation of $[\{S_{loc}, S_{loc}\}]_4$ was done in [9] as

$$\begin{aligned}\frac{1}{\sqrt{G}}\left[\{S_{loc}, S_{loc}\}\right]_4 = &-\frac{W}{2(d-1)}\left((d-4)X - \frac{dl^3}{4(d-1)(d-2)^2}\right)R^2 \\ &-\frac{W}{2(d-1)}\left((d-4)Y + \frac{l^3}{(d-2)^2}\right)R_{\mu\nu}R^{\mu\nu} - \frac{d-4}{2(d-1)}WZR_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma} \\ &+\left(2X + \frac{d}{2(d-1)}Y + \frac{2}{d-1}Z\right).\end{aligned} \tag{18}$$

For $d \geq 6$, from $[\{S_{loc}, S_{loc}\}]_4 = 0$ one finds

$$X = \frac{dl^3}{4(d-1)(d-2)^2(d-4)}, \quad Y = -\frac{l^3}{(d-2)^2(d-4)}, \quad Z = 0. \tag{19}$$

Using them, one can calculate $[\{S_{loc}, S_{loc}\}]_6$ as [9]

$$\begin{aligned}\frac{1}{\sqrt{G}}\left[\{S_{loc}, S_{loc}\}\right]_6 = &\ \Phi\left[\left(-4X + \frac{d+2}{2(d-1)}Y\right)RR_{\mu\nu}R^{\mu\nu} + \frac{d+2}{2(d-1)}XR^3\right. \\ &-4YR^{\mu\lambda}R^{\nu\sigma}R_{\mu\nu\lambda\sigma} + (4X+2Y)R^{\mu\nu}\nabla_\mu\nabla_\nu R - 2YR^{\mu\nu}\nabla^2 R_{\mu\nu} \\ &\left.+\left(-2X - \frac{d-2}{2(d-1)}Y\right)R\nabla^2 R\right] + (\text{contributions from } [\mathcal{L}_{loc}]_6) \\ = &\ l^4\left[-\frac{3d+2}{2(d-1)(d-2)^2(d-4)}RR_{\mu\nu}R^{\mu\nu} + \frac{d(d+2)}{8(d-1)^2(d-2)^3(d-4)}R^3\right. \\ &+\frac{4}{(d-2)^3(d-4)}R^{\mu\lambda}R^{\nu\sigma}R_{\mu\nu\lambda\sigma} - \frac{1}{(d-1)(d-2)^2(d-4)}R^{\mu\nu}\nabla_\mu\nabla_\nu R\end{aligned}$$

$$+\frac{2}{(d-2)^3(d-4)}R^{\mu\nu}\nabla^2 R_{\mu\nu}-\frac{1}{(d-1)(d-2)^3(d-4)}R\nabla^2 R\Big]$$
$$+(\text{contributions from } [\mathcal{L}_{loc}]_6). \tag{20}$$

The weight d flow equation (13), which is related with the conformal anomaly in d dimensions [1, 9], is written by

$$-\frac{W}{2(d-1)}\frac{1}{\sqrt{G}}G_{\mu\nu}\frac{\delta\Gamma}{\delta G_{\mu\nu}}=-\left[\{S_{loc},S_{loc}\}\right]_d. \tag{21}$$

This $G_{\mu\nu}\frac{\delta\Gamma}{\delta G_{\mu\nu}}$ can be regarded as the sum of conformal anomaly $\mathcal{W}_d$ and the total derivative term $\nabla_\mu\mathcal{J}^\mu_d$ in d dimensions. Thus we rewrite (21) as following

$$\kappa^2\mathcal{W}_d+\nabla_\mu\mathcal{J}^\mu_d=\frac{d-1}{W\sqrt{G}}\left[\{S_{loc},S_{loc}\}\right]_d. \tag{22}$$

Here κ^2 is $d+1$ dimensional gravitational coupling. Using the above relation, one can get the holographic conformal anomaly in 4 dimensions from (18):

$$\kappa^2\mathcal{W}_4=-\frac{l}{2\sqrt{G}}\left[\{S_{loc},S_{loc}\}\right]_4=l^3\left(\frac{1}{24}R^2-\frac{1}{8}R_{\mu\nu}R^{\mu\nu}\right). \tag{23}$$

This agrees with the result in Refs.[6] calculated by various methods (using AdS/CFT duality). Further, the above calculation can be extended to include dilaton (a scalar). Note that holographic conformal anomaly for the presence of dilaton turns out to be scheme-dependent [3]. The conformal anomaly in 6 dimensions is calculated from (20) as

$$\begin{aligned}\kappa^2\mathcal{W}_6 &= -\frac{l}{2\sqrt{G}}\left[\{S_{loc},S_{loc}\}\right]_6\\ &= l^5\Big(\frac{1}{128}RR_{\mu\nu}R^{\mu\nu}-\frac{3}{3200}R^3-\frac{1}{64}R^{\mu\lambda}R^{\nu\sigma}R_{\mu\nu\lambda\sigma}\\ &\qquad+\frac{1}{320}R^{\mu\nu}\nabla_\mu\nabla_\nu R-\frac{1}{128}R^{\mu\nu}\nabla^2R_{\mu\nu}+\frac{1}{1280}R\nabla^2R\Big),\end{aligned} \tag{24}$$

which coincides exactly with 6 dimensional conformal anomaly in [6]. Above discussions have already been performed in Ref.[9].

The local terms of weight 6: $[\mathcal{L}_{loc}]_6$ is assumed to be

$$[\mathcal{L}_{loc}]_6 = aR^3 + bRR_{\mu\nu}R^{\mu\nu} + cRR_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma} + eR_{\mu\nu\lambda\sigma}R^{\mu\rho}R^{\nu\sigma} \\ +f\nabla_\mu R\nabla^\mu R + g\nabla_\mu R_{\nu\rho}\nabla^\mu R^{\nu\rho} + h\nabla_\mu R_{\nu\rho\sigma\tau}\nabla^\mu R^{\nu\rho\sigma\tau} + jR^{\mu\nu}R^\rho_\nu R_{\rho\mu}. \tag{25}$$

Adding above terms to (20), we obtain

$$\begin{aligned}
\frac{1}{\sqrt{G}}\left[\{S_{loc}, S_{loc}\}\right]_6 = &\left(b\left(\frac{d}{2}-3\right)\frac{2}{l} - \frac{(3d+2)l^4}{2(d-1)(d-2)^3(d-4)}\right)RR_{\mu\nu}R^{\mu\nu} \\
&+\left(a\left(\frac{d}{2}-3\right)\frac{2}{l} + \frac{d(d+2)l^4}{8(d-1)^2(d-2)^3(d-4)}\right)R^3 \\
&+\left(\left\{-e\left(\frac{d}{2}+2\right) - 2g - \frac{3j}{2}(2-d)\right\}\frac{2}{l} + \frac{4l^4}{(d-2)^3(d-4)}\right)R^{\mu\lambda}R^{\nu\sigma}R_{\mu\nu\lambda\sigma} \\
&+\left(\left\{b(2-d) - 4c + e\left(\frac{d}{2}-1\right) + \frac{3j}{2}(2-d)\right\}\frac{2}{l}\right. \\
&\left.-\frac{l^4}{(d-1)(d-2)^2(d-4)}\right)R^{\mu\nu}\nabla_\mu\nabla_\nu R \\
&+\left(\{2b(1-d) - de + 2g - 3j\}\frac{2}{l} + \frac{2l^4}{(d-2)^3(d-4)}\right)R^{\mu\nu}\nabla^2 R_{\mu\nu} \\
&+\left(\left\{6a(1-d) - b\left(1+\frac{d}{2}\right) - 2c - \frac{1}{2}e + 2f\right\}\frac{2}{l}\right. \\
&\left.-\frac{l^4}{(d-1)(d-2)^3(d-4)}\right)R\nabla^2 R \\
&+\left(\frac{d}{2}g + 2h + 2f(d-1)\right)\frac{2}{l}\nabla^4 R + \left(\frac{d}{2}-3\right)\frac{2c}{l}RR_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \\
&+\left(6a(1-d) - db - 4c - \frac{3}{4}e + \left(\frac{d}{2}-1\right)f - \frac{g}{2} + \frac{3}{8}(2-d)j\right)\frac{2}{l}\nabla_\mu R\nabla^\mu R \\
&+\left(2b(1-d) + 2e(1-d) + g\left(\frac{d}{2}-1\right) - 8h - 3j\right)\frac{2}{l}\nabla_\kappa R_{\mu\nu}\nabla^\kappa R^{\mu\nu} \\
&+\left((2d-3)e + 2g + 8h + \frac{3}{2}(2-d)j\right)\frac{2}{l}\nabla_\kappa R^{\mu\nu}\nabla_\nu R^\kappa_\mu \\
&+((d-1)e + 2g - dj)\frac{2}{l}R^{\mu\nu}R^\rho_\nu R_{\mu\rho} + (2-d)\frac{2e}{l}R^{\mu\nu\rho\sigma}\nabla_\mu\nabla_\rho R_{\nu\sigma}
\end{aligned}$$

$$
\begin{aligned}
&+\left(2c(1-d)+\left(\frac{d}{2}-1\right)h\right)\frac{2}{l}\nabla_\alpha R_{\mu\nu\rho\sigma}\nabla^\alpha R^{\mu\nu\rho\sigma}\\
&+(2c(1-d)+2h)\frac{2}{l}R_{\mu\nu\rho\sigma}\nabla^2 R^{\mu\nu\rho\sigma}\\
&+\left(4R^{\mu\rho\sigma\tau}R_{\lambda\mu}R^{\lambda}_{\rho\sigma\tau}-4R^{\mu\rho\sigma\tau}R^{\nu}_{\mu\lambda\rho}R^{\lambda}_{\nu\tau\sigma}\right)\frac{2h}{l}\\
&+\left(-8R^{\mu\rho\sigma\tau}R^{\nu}_{\mu\lambda\sigma}R^{\lambda}_{\tau\nu\rho}+4\nabla_\nu R_{\mu\rho\sigma\tau}\nabla^\mu R^{\nu\rho\sigma\tau}\right)\frac{2h}{l}\\
=&\Bigg[\left(\frac{(d-6)b}{l}-\frac{(3d+2)l^4}{2(d-1)(d-2)^2(d-4)}\right)RR_{\mu\nu}R^{\mu\nu}\\
&+\left(\frac{(d-6)a}{l}+\frac{d(d+2)l^4}{8(d-1)^2(d-2)^3(d-4)}\right)R^3+\left(\frac{(d-6)c}{l}\right)RR_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}\\
&+\left(\frac{(d-6)e}{l}+\frac{16h}{l}+\frac{4l^4}{(d-2)^3(d-4)}\right)R^{\mu\lambda}R^{\nu\sigma}R_{\mu\nu\lambda\sigma}\\
&+\left(-\left(3-\frac{d}{2}\right)\frac{2f}{l}+\frac{4h}{l}+\frac{dl^4}{2(d-1)(d-2)^3(d-4)}\right)\nabla^\mu R\nabla_\mu R\\
&+\left(\frac{(d-6)g}{l}-\frac{2l^4}{(d-2)^3(d-4)}+\frac{16h}{l}\right)\nabla^\rho R^{\mu\nu}\nabla_\rho R_{\mu\nu}\\
&+\left(\frac{2j}{l}\left(\frac{d}{2}-3\right)-\frac{16h}{l}\right)R^{\mu\nu}R^{\rho}_{\nu}R_{\rho\mu}\\
&+\left(\frac{(6-d)h}{l}\right)R_{\mu\nu\lambda\sigma}\nabla^2 R^{\mu\nu\lambda\sigma}\Bigg]+\text{total derivative terms}\ .
\end{aligned}
\tag{26}
$$

For $d\geq 8$, from $[\{S_{loc},S_{loc}\}]_6=0$, if one neglects the total derivative terms, the coefficients a,b,c,e,f,g,h,j are

$$
\begin{aligned}
&a=-\frac{d(d+2)l^5}{8(d-1)^2(d-2)^3(d-4)(d-6)}\\
&b=\frac{(3d+2)l^5}{2(d-1)(d-2)^2(d-4)(d-6)}\ ,\quad c=0\\
&e=-\frac{4l^5}{(d-2)^3(d-4)(d-6)}\ ,\quad f=-\frac{dl^5}{2(d-1)(d-2)^3(d-4)(d-6)}\\
&g=\frac{2l^5}{(d-2)^3(d-4)(d-6)}\ ,\quad h=0\ ,\quad j=0.
\end{aligned}
\tag{27}
$$

We can also consider $d=8$ case in the same way. In $d=8$ case, one obtains $\frac{1}{\sqrt{G}}[\{S_{loc}, S_{loc}\}]_8$ as follows

$$
\begin{aligned}
\frac{1}{\sqrt{G}}[\{S_{loc}, S_{loc}\}]_8 =& \left(-\frac{(d+8)X^2}{4(d-1)} + \frac{(d+4)al}{2(d-1)(d-2)}\right) R^4 \\
&+ \left(2X^2 + \frac{(-d+4)XY}{2(d-1)} - \frac{6al}{(d-2)} + \frac{(4-d)bl}{2(d-1)(d-2)} + \frac{el}{2(d-1)(d-2)}\right. \\
&\left. - \frac{2fl}{(d-1)(d-2)}\right) R^2\nabla^2 R + \left(-\frac{(d+8)Y^2}{4(d-1)} - \frac{2bl}{d-2}\right)(R^{\mu\nu}R_{\mu\nu})^2 \\
&+ \left(-4X^2 - \frac{d}{4(d-1)}Y^2 - 2XY\right)(\nabla^2 R)^2 \\
&+ \left(4X^2 - \frac{(d+8)XY}{2(d-1)} - \frac{6al}{(d-2)} + \frac{(d+4)bl}{2(d-1)(d-2)}\right) R^2 R^{\mu\nu}R_{\mu\nu} \\
&+ \left(4X^2 + Y^2 + 4XY\right)\nabla^\mu\nabla^\nu R \nabla_\mu\nabla_\nu R \\
&+ \left(-8X^2 - 4XY + \frac{12al}{d-2} + \frac{bl}{d-1} + \frac{del}{2(d-1)(d-2)}\right) R R^{\mu\nu}\nabla_\mu\nabla_\nu R \\
&+ \left(\frac{(d-4)(-2d+1)Y^2}{2(d-1)} + 2XY - \frac{2bl}{d-2} - \frac{el}{d-2} + \frac{4fl}{d-2}\right) R^{\mu\nu}R_{\mu\nu}\nabla^2 R \\
&+ \left(-2Y^2 - 4XY\right)\nabla^2 R^{\mu\nu}\nabla_\mu\nabla_\nu R + \left(4Y^2 - \frac{2el}{d-2} + \frac{4gl}{d-2}\right)\nabla^2 R^{\mu\nu} R_{\mu\lambda\nu\kappa}R^{\lambda\kappa} \\
&+ Y^2\nabla^2 R_{\mu\nu}\nabla^2 R^{\mu\nu} + \left(4Y^2 + \frac{4el}{d-2}\right) R^\lambda_{\mu\kappa\nu} R^{\mu\nu} R^\kappa_{\sigma\lambda\gamma} R^{\sigma\gamma} \\
&+ \left(-4Y^2 - 8XY\right) R^\lambda_{\mu\kappa\nu} R^{\mu\nu}\nabla_\lambda\nabla^\kappa R + \frac{4el}{d-2} R_{\kappa\lambda} R^\kappa_{\sigma\mu\nu}\nabla^\mu\nabla^\lambda R^{\nu\sigma} \\
&+ \left(8XY - \frac{4bl}{d-2} + \frac{(-d+6)el}{2(d-1)(d-2)} + \frac{2gl}{(d-1)(d-2)}\right) R^\kappa_\lambda R^\lambda_{\mu\kappa\nu} R^{\mu\nu} R \\
&+ \left(4XY - \frac{4bl}{(d-2)} - \frac{el}{(d-1)} - \frac{2gl}{(d-1)(d-2)}\right) R_{\kappa\lambda} R \nabla^2 R^{\kappa\lambda} \\
&+ \left(-\frac{6al}{d-2} - \frac{bl}{d-1} + \frac{3el}{4(d-1)(d-2)} + \frac{dfl}{2(d-1)(d-2)}\right. \\
&\left. + \frac{gl}{2(d-1)(d-2)}\right) R(\nabla R)^2 + \frac{l}{d-2}\left(4b + 2e + 2g\right) R_{\kappa\lambda} R^{\mu\kappa}\nabla_\mu\nabla^\lambda R
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{l}{d-2}\left(12a+2b+\frac{e}{2}-2f\right)R_{\mu\nu}\nabla^\mu R\nabla^\nu R \\
&+\frac{l}{d-2}\left(-2b-2e+\frac{dg}{2(d-1)}\right)R\nabla_\kappa R_{\mu\nu}\nabla^\kappa R^{\mu\nu}+\left(\frac{2fl}{d-2}+\frac{gl}{2(d-1)}\right)R\nabla^4 R \\
&-\frac{(4f+2g)l}{d-2}R_{\mu\nu}\nabla^\mu\nabla^\nu\nabla^2 R+\frac{(4b+2e)l}{d-2}R_{\kappa\lambda}R^{\mu\nu}\nabla^\kappa\nabla^\lambda R_{\mu\nu} \\
&+\frac{l}{d-2}\left(4b+4e-2g\right)R_{\kappa\lambda}\nabla^\kappa R_{\mu\nu}\nabla^\lambda R^{\mu\nu}+\frac{l}{d-2}\left(4b-2g\right)R_{\kappa\lambda}\nabla_\mu R\nabla^\lambda R^{\mu\kappa} \\
&-\frac{l}{d-2}\left(4b+2e\right)R_{\kappa\lambda}\nabla_\mu R\nabla^\mu R^{\kappa\lambda}+\frac{l}{d-2}\left(e-\frac{2g}{d-1}\right)RR^{\mu\nu}R_{\kappa\mu}R^{\kappa\nu} \\
&+\frac{l}{(d-1)(d-2)}\left((2d-1)e-2g\right)R\nabla^\mu R^{\nu\kappa}\nabla_\nu R_{\mu\kappa} \\
&-\frac{del}{(d-1)(d-2)}RR^{\mu\nu\kappa\lambda}\nabla_\mu\nabla_\kappa R_{\nu\lambda}+\frac{2el}{d-2}R_{\mu\nu}\nabla_\kappa R^{\nu\rho}\nabla_\rho R^{\kappa\mu} \\
&-\frac{4(e+g)l}{d-2}R_{\kappa\lambda}R^\lambda_{\nu\rho\mu}R^{\kappa\rho}R^{\nu\mu}-\frac{8gl}{d-2}R_{\kappa\lambda}\nabla_\mu R^{\lambda\nu}\nabla^\mu R^\kappa_\nu-\frac{4gl}{d-2}R_{\kappa\lambda}R^\kappa_\nu\nabla^2 R^{\lambda\nu} \\
&-\frac{4(e+g)l}{d-2}R_{\kappa\lambda}R^\mu_\nu\nabla_\mu\nabla^\lambda R^{\kappa\nu}-\frac{4(e-g)l}{d-2}R_{\kappa\lambda}\nabla_\mu R^{\kappa\nu}\nabla^\lambda R^\mu_\nu \\
&-\frac{(2e-4g)l}{d-2}R^\lambda_\kappa R^{\mu\nu}\nabla^2 R^\kappa_{\mu\lambda\nu}+\frac{2gl}{d-2}R_{\mu\nu}\nabla^4 R^{\mu\nu} \\
&+\frac{4el}{d-2}R_{\kappa\lambda}\nabla^\mu R^{\nu\sigma}\nabla^\lambda R^\kappa_{\sigma\mu\nu}-\frac{(4e-8g)l}{d-2}R^{\kappa\lambda}\nabla^\mu R^\nu_\sigma\nabla_\mu R^\sigma_{\kappa\nu\lambda} \qquad (28) \\
&+\frac{(2e+4g)l}{d-2}R_{\kappa\lambda}R^\kappa_\rho R^\rho_\nu R^{\nu\lambda}+\frac{4el}{d-2}\left(R_{\kappa\lambda}R^\nu_\sigma R^{\rho\sigma\lambda\mu}R^\kappa_{\rho\mu\nu}+R_{\kappa\lambda}R^\sigma_\nu R^{\rho\nu\lambda\mu}R^\kappa_{\sigma\mu\rho}\right) .
\end{aligned}
$$

Substituting X, Y, a, b, e, f, g, j in (19) and (27) into the above equation and putting $d = 8$, we obtain the explicit form of $[\{S_{loc}, S_{loc}\}]_8$ and conformal anomaly in 8 dimensions

$$
\begin{aligned}
&-\frac{2}{l^7}\kappa^2\mathcal{W}_8=\frac{1}{l^6\sqrt{G}}[\{S_{loc},S_{loc}\}]_8 \\
&\quad=\frac{13}{889056}R^4-\frac{1}{2352}R^2\nabla^2 R-\frac{79}{36288}(R^{\mu\nu}R_{\mu\nu})^2-\frac{1}{508032}(\nabla^2 R)^2 \\
&\quad+\frac{53}{63504}R^2R^{\mu\nu}R_{\mu\nu}+\frac{1}{112896}\nabla^\mu\nabla^\nu R\nabla_\mu\nabla_\nu R+\frac{61}{63504}RR^{\mu\nu}\nabla_\mu\nabla_\nu R \\
&\quad-\frac{23}{10368}R^{\mu\nu}R_{\mu\nu}\nabla^2 R-\frac{1}{24192}\nabla^2 R^{\mu\nu}\nabla_\mu\nabla_\nu R+\frac{1}{576}\nabla^2 R^{\mu\nu}R_{\mu\lambda\nu\kappa}R^{\lambda\kappa}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{20736}\nabla^2 R_{\mu\nu}\nabla^2 R^{\mu\nu}-\frac{7}{5184}R^{\lambda}_{\mu\kappa\nu}R^{\mu\nu}R^{\kappa}_{\sigma\lambda\gamma}R^{\sigma\gamma}-\frac{1}{12096}R^{\lambda}_{\mu\kappa\nu}R^{\mu\nu}\nabla_\lambda\nabla^\kappa R\\
&-\frac{1}{648}R_{\kappa\lambda}R^{\kappa}_{\sigma\mu\nu}\nabla^\mu\nabla^\lambda R^{\nu\sigma}-\frac{13}{3024}R^{\kappa}_{\lambda}R^{\lambda}_{\mu\kappa\nu}R^{\mu\nu}R-\frac{37}{9072}R_{\kappa\lambda}R\nabla^2 R^{\kappa\lambda}\\
&-\frac{31}{28224}R(\nabla R)^2+\frac{71}{18144}R_{\kappa\lambda}R^{\mu\kappa}\nabla_\mu\nabla^\lambda R+\frac{65}{28224}R_{\mu\nu}\nabla^\mu R\nabla^\nu R\\
&-\frac{23}{18144}R\nabla_\kappa R_{\mu\nu}\nabla^\kappa R^{\mu\nu}-\frac{1}{72576}R\nabla^4 R-\frac{1}{6048}R_{\mu\nu}\nabla^\mu\nabla^\nu\nabla^2 R\\
&+\frac{2}{567}R_{\kappa\lambda}R^{\mu\nu}\nabla^\kappa\nabla^\lambda R_{\mu\nu}+\frac{43}{18144}R_{\kappa\lambda}\nabla^\kappa R_{\mu\nu}\nabla^\lambda R^{\mu\nu}\\
&+\frac{71}{18144}R_{\kappa\lambda}\nabla_\mu R\nabla^\lambda R^{\mu\kappa}-\frac{2}{567}R_{\kappa\lambda}\nabla_\mu R\nabla^\mu R^{\kappa\lambda}-\frac{1}{2268}RR^{\mu\nu}R_{\kappa\mu}R^{\kappa\nu}\\
&-\frac{1}{1134}R\nabla^\mu R^{\nu\kappa}\nabla_\nu R_{\mu\kappa}+\frac{1}{2268}RR^{\mu\nu\kappa\lambda}\nabla_\mu\nabla_\kappa R_{\nu\lambda}\\
&-\frac{1}{1296}R_{\mu\nu}\nabla_\kappa R^{\nu\rho}\nabla_\rho R^{\kappa\mu}+\frac{1}{1296}R_{\kappa\lambda}R^{\lambda}_{\nu\rho\mu}R^{\kappa\rho}R^{\nu\mu}\\
&-\frac{1}{648}R_{\kappa\lambda}\nabla_\mu R^{\lambda\nu}\nabla^\mu R^{\kappa}_{\nu}-\frac{1}{1296}R_{\kappa\lambda}R^{\kappa}_{\nu}\nabla^2 R^{\lambda\nu}+\frac{1}{1296}R_{\kappa\lambda}R^{\mu}_{\nu}\nabla_\mu\nabla^\lambda R^{\kappa\nu}\\
&+\frac{1}{432}R_{\kappa\lambda}\nabla_\mu R^{\kappa\nu}\nabla^\lambda R^{\mu}_{\nu}+\frac{1}{648}+R^{\lambda}_{\kappa}R^{\mu\nu}\nabla^2 R^{\kappa}_{\mu\lambda\nu}+\frac{1}{2592}R_{\mu\nu}\nabla^4 R^{\mu\nu}\\
&-\frac{1}{648}R_{\kappa\lambda}\nabla^\mu R^{\nu\sigma}\nabla^\lambda R^{\kappa}_{\sigma\mu\nu}+\frac{1}{324}R^{\kappa\lambda}\nabla^\mu R^{\nu}_{\sigma}\nabla_\mu R^{\sigma}_{\kappa\nu\lambda}\\
&-\frac{1}{648}\left(R_{\kappa\lambda}R^{\nu}_{\sigma}R^{\rho\sigma\lambda\mu}R^{\kappa}_{\rho\mu\nu}+R_{\kappa\lambda}R^{\sigma}_{\nu}R^{\rho\nu\lambda\mu}R^{\kappa}_{\sigma\mu\rho}\right).
\end{aligned}
\tag{29}
$$

As one can see already in eight dimensions (and omitting total derivative terms) the explicit result for holographic conformal anomaly is quite complicated. It is clear that going to higher dimensions it is getting much more complicated.

As an example, we consider de Sitter space, where curvatures are covariantly constant and given by

$$
R_{\mu\nu\rho\sigma}=\frac{1}{l^2}\left(g_{\mu\rho}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\rho}\right)\ ,\quad R_{\mu\nu}=\frac{d-1}{l^2}g_{\mu\nu}\ ,\quad R=\frac{d(d-1)}{l^2}\ . \tag{30}
$$

Here l is the radius of the de Sitter space[7] and it is related to the cosmological constant Λ by $\Lambda=\frac{(d-2)(d-1)}{l^2}$. By putting $d=8$ in (30) and substituting the

[7]The choice of de Sitter space is caused by several reasons. First, the explicit expression for holographic d8 CA is significally simplified on this highly symmetric background.

curvatures into (29), we find an expression for the anomaly:

$$\kappa^2 \mathcal{W}_8 = -\frac{l}{2\sqrt{G}}[\{S_{loc}, S_{loc}\}]_8 = -\frac{62069}{1296l} \ . \tag{31}$$

We should note that $\frac{1}{\kappa^2}$ is 9 dimensional one here, then κ^2 has the dimension of 7th power of the length.

In refs.[11, 12] the QFT conformal anomalies coming from scalar and spinor fields in 8d de Sitter space are found

$$T_{\text{scalar}} = -\frac{23}{34560\pi^4 l^8} \ , \quad T_{\text{spinor}} = -\frac{2497}{34560\pi^4 l^8} \ . \tag{32}$$

If there is supersymmetry, the number of the scalars is related with that of the spinors. For example, consider the matter supermultiplet and take only scalar-spinor part of it (one real scalar and one Dirac spinor) as vector is not conformally invariant in d8 dimensions. If there is N^4 pairs of scalars and spinors[8] , the total anomaly should be given by

$$\mathcal{W}_8 = N^4 \left(T_{\text{scalar}} + T_{\text{spinor}}\right) = -\frac{7N^4}{6\left(2\pi\right)^4 l^8} \ . \tag{33}$$

By comparing (33) with (31), we find

$$\frac{1}{\kappa^2} = \frac{216N^4}{8867\left(2\pi\right)^4 l^7} \ , \tag{34}$$

which might be useful to establish the proposal for $\text{AdS}_9/\text{CFT}_8$.

Second, the quantum field calculation of d8 conformal anomaly has been done so far only for de Sitter background as we mention in the text. Third, even in d4 case the calculation of CA on de Sitter space has been done by various authors because it may have the interesting applications in cosmology/BH physics and it is related with Casimir energy.

[8] In $\text{AdS}_5/\text{CFT}_4$ correspondence where exact CFT is maximally SUSY $SU(N)$ super Yang-Mills theory the supermultiplet structure gives the multiplier N^2 in front of QFT CA which should be compared with holographic one. The $\text{AdS}_3/\text{CFT}_2$ predicts the correspondent multiplier to be N. In $\text{AdS}_7/\text{CFT}_6$ correspondence where recently constructed (0,2) tensor multiplet plays the role of exact CFT the supermultiplet structure predicts the multiplier N^3 in front of QFT CA which should be compared with holographic one. It is very natural to expect then that $\text{AdS}_9/\text{CFT}_8$ correspondence exists (as well as respective d8 exact CFT) and the number of fields in CA appears with the factor N^4. As far as we know several attempts to construct d8 exact CFT are on the way currently.

Of course, the above relation gives only the indication (the numerical factor is definitely wrong) as we considered only scalar-spinor part of non-conformal multiplet. On the same time it is known that for AdS_7/CFT_6 correspondence the tensor multiplet gives brane CFT while for d4 the gauge fields play the important role (super Yang-Mills theory). As far as we know the rigorous proposal for d8 brane CFT does not exist yet. However, it is evident that not only scalars and spinors but also other fields will be part of d8 CFT. It would be extremely interesting to construct the candidate for such theory. Then the above d8 holographic anomaly may be used to check the correctness of such proposal.

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